AIAA JOURNAL
Vol. 48, No. 6, June 2010

Technical Notes

Improved Drag Correlation for Spheres and Application to Shock-Tube Experiments

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DOI: 10.2514/1.J050161

Nomenclature

a	=	speed of sound, m/s
C_D	=	drag coefficient
C_1, C_2, C_3	=	coefficients in curve fit
d^p	=	particle diameter, m
F	=	drag force, N
f_1, f_2, f_3	=	functions in curve fit
Kn	=	Knudsen number
M	=	relative Mach number
m^p	=	mass of particle, kg
Re	=	relative Reynolds number
t	=	time, s
u	=	velocity, m/s
γ	=	ratio of specific heats
μ^g	=	dynamic viscosity, kg/(ms)
ν^g	=	kinematic viscosity, m^2/s
ξ	=	functions
ρ	=	density, kg/m^3
τ_s	=	nondimensional time

Subscripts

cr	=	critical (Mach number)
s	=	shock
std	=	standard (drag coefficient)
sub	=	subsonic
sup	=	supersonic
2	=	behind shock wave

Superscripts

g	=	gas
p	=	particle

I. Introduction

EMPIRICAL correlations for the quasi-steady drag coefficient of a sphere in compressible flow have been presented by several authors (e.g., Henderson [1] and Loth [2]). Such correlations are needed in numerical simulations of compressible multiphase flows involving spherical particles. In this Note, the accuracy of the correlations of Henderson [1] and Loth [2] are assessed using the data collected by Bailey and Starr [3], and an improved correlation for the drag coefficient of a sphere in compressible flow is developed. The improved correlation is validated for shock-particle interaction, using the recent shock-tube experiments of Jourdan et al. [4].

II. Improved Drag-Coefficient Correlation

The correlations of Henderson [1] and Loth [2] are compared with the data of Bailey and Starr [3] in Fig. 1. For a given Mach number, Henderson's [1] correlation decreases monotonically with increasing an Reynolds number and thus fails to capture the rise in the drag coefficient as the critical Reynolds number is approached. Loth's [2] correlation shows a consistent early rise as the Reynolds number increases for a given Mach number.§ Furthermore, there is an overlap in Loth's [2] correlation for $0.89 \leq M \leq 1.0$ that does not exist in the experimental data. For Henderson's [1] correlation, the largest deviation from the data of Bailey and Starr [3] is about 16% for Reynolds numbers below 10^4. For Loth's [2] correlation, the largest error is about 55%, concentrated around a Mach number of about 0.9. Discrepancies of this magnitude call for the development of an improved drag-coefficient correlation.

The improved correlation is based on the following assumptions:

1) Attention is limited to continuum flows, i.e., we assume that the Knudsen number $Kn = \sqrt{\gamma\pi/2}M/Re < 0.01$, where γ is the ratio of specific heats, M is the Mach number based on the relative velocity (the velocity of the particle relative to that of the fluid), and Re is the Reynolds number based on the relative velocity and particle diameter.

2) The particle temperature is constant and equal to the surrounding gas temperature.

3) In the limit of zero Mach number, the correlation should approach the standard-drag correlation of Clift and Gauvin [5]:

$$C_{D,\mathrm{std}}(Re) = \frac{24}{Re}(1 + 0.15Re^{0.687}) + 0.42\left(1 + \frac{42500}{Re^{1.16}}\right)^{-1} \tag{1}$$

4) Attention is limited to subcritical Reynolds numbers, i.e., $Re \lessapprox 2 \cdot 10^5$, above which the attached boundary layer becomes turbulent.

5) We focus on the range $0 \leqslant M \leqslant 1.75$ and make use of the data compiled by Bailey and Starr [3]. The data of Bailey and Starr is used because it appears to be the most comprehensive compilation of accurate drag-coefficient measurements.

The resulting improved drag-coefficient correlation consists of three separate correlations for subcritical ($M \leqslant M_{\mathrm{cr}} \approx 0.6$), supersonic ($1 < M \leqslant 1.75$), and intermediate ($M_{\mathrm{cr}} < M \leqslant 1$) Mach number regimes:

$$C_D(Re, M) = \begin{cases} C_{D,\mathrm{std}}(Re) + [C_{D,M_{\mathrm{cr}}}(Re) - C_{D,\mathrm{std}}(Re)]\frac{M}{M_{\mathrm{cr}}} & \text{if } M \leqslant M_{\mathrm{cr}} \\ C_{D,\mathrm{sub}}(Re, M) & \text{if } M_{\mathrm{cr}} < M \leqslant 1.0 \\ C_{D,\mathrm{sup}}(Re, M) & \text{if } 1.0 < M \leqslant 1.75 \end{cases} \tag{2}$$

This separation is motivated by the following observations.

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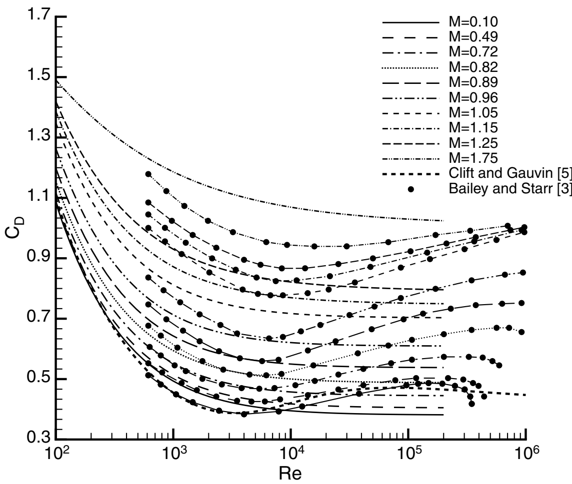
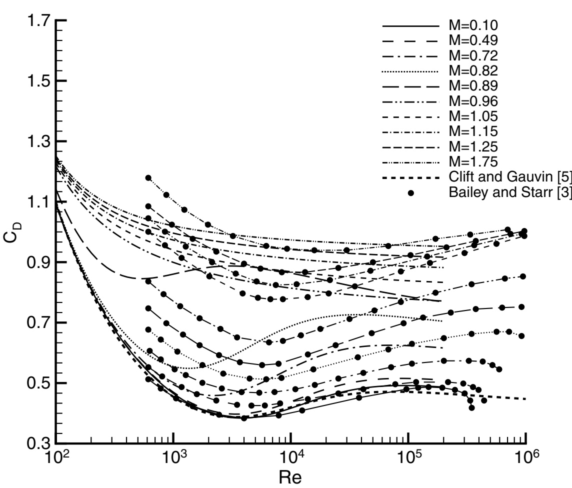
§It should be noted that equation (25b) in Loth's article [2] is missing the term $2\sqrt{\pi}/3s$.



a) Correlation of Henderson [1]



b) Correlation of Loth [2]

Fig. 1 Comparison of drag-coefficient correlations with data of Bailey and Starr [3].

1) For subcritical Mach numbers, the flow around a spherical particle is shock free; thus, the drag coefficient is only weakly affected by compressibility effects.

2) For supercritical but subsonic Mach numbers, a shock wave of limited radial extent exists on the sphere, and the drag coefficient becomes more strongly dependent on the Mach number.

3) For supersonic Mach numbers, a bow shock exists that leads to a large increase of the drag coefficient. (The bow shock does not appear at precisely sonic conditions, but the preceding simple separation into regimes is sufficient for our modeling purposes.)

4) The upper limit of $M = 1.75$ is used, because that is the maximum Mach number for which Bailey and Starr [3] presented data. This limit is sufficient for our purposes, because we are mainly interested in unsteady shock waves accelerating initially stationary particles. For such interactions, the largest possible Mach number is $M \approx 1.89$ for $\gamma = 1.4$. We take $M_{\rm cr} = 0.6$ in the following for simplicity.

In the subcritical regime, the drag coefficient is expressed as a linear interpolation between the drag coefficients at $M = 0$ and $M = M_{\rm cr}$. (It should be noted that we do not make use of the data of Bailey and Starr [3] for $M < 0.6$, because it exhibits drag coefficients that lie below the standard-drag curve.) Based on the functional form of Eq. (1), $C_{D,M_{\rm cr}}(Re)$ is expressed as

$$C_{D,M_{\rm cr}}(Re) = \frac{24}{Re}(1 + 0.15Re^{0.684}) + 0.513\left(1 + \frac{483}{Re^{0.669}}\right)^{-1} \quad (3)$$

Note the similarity of the coefficients when compared with Eq. (1) due to the weak influence of the compressibility in this regime, as stated previously.

In the supersonic regime, the drag coefficient is expressed as a nonlinear interpolation between the drag coefficients at $M = 1$ and $M = 1.75$:

$$C_{D,\sup}(Re, M) = C_{D,M=1}(Re) + [C_{D,M=1.75}(Re) - C_{D,M=1}(Re)]\xi_{\sup}(M, Re) \quad (4)$$

where

$$C_{D,M=1}(Re) = \frac{24}{Re}(1 + 0.118Re^{0.813}) + 0.69\left(1 + \frac{3550}{Re^{0.793}}\right)^{-1} \quad (5)$$

$$C_{D,M=1.75}(Re) = \frac{24}{Re}(1 + 0.107Re^{0.867}) + 0.646\left(1 + \frac{861}{Re^{0.634}}\right)^{-1} \quad (6)$$

and

$$\xi_{\sup}(M, Re) = \sum_{i=1}^{3} f_{i,\sup}(M) \prod_{\substack{j\neq i\\ j=1}}^{3} \frac{\log Re - C_{j,\sup}}{C_{i,\sup} - C_{j,\sup}} \quad (7)$$

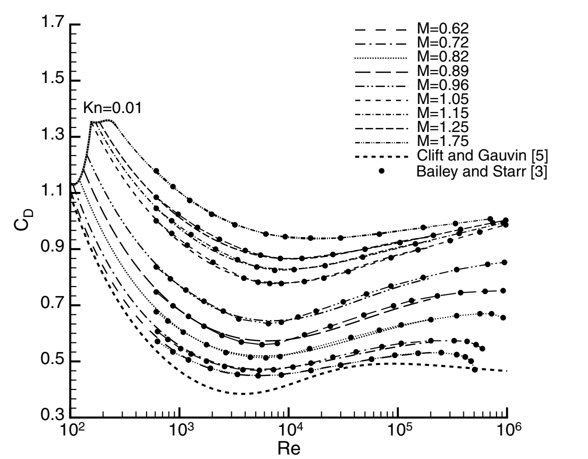


Fig. 2 Comparison of new drag-coefficient correlation with data of Bailey and Starr [3].

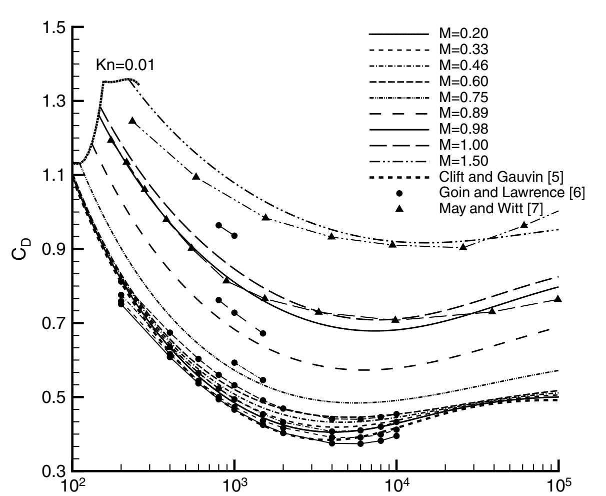


Fig. 3 Comparison of new drag-coefficient correlation with data of Goin and Lawrence [6] and May and Witt [7].

Table 1 Summary of cases from Jourdan et al. [4] used to validate new correlation.

Case	Gases	M	Re	d^p, mm	ρ^p, kg/m^3
148	Air/air	0.47–0.66	40,000–56,000	1.92	1130
166a	Air/air	0.47–0.61	37,000–48,000	1.96	1204
181a	Helium/air	1.28–1.60	1450–1900	0.62	1096
184a	Helium/air	0.70–1.53	3000–6000	1.60	25

with

$$f_{1,\mathrm{sup}}(M) = -2.963 + 4.392M - 1.169M^2 - 0.027M^3 - 0.233\exp[(1-M)/0.011] \quad (8)$$

$$f_{2,\mathrm{sup}}(M) = -6.617 + 12.11M - 6.501M^2 + 1.182M^3 - 0.174\exp[(1-M)/0.01] \quad (9)$$

$$f_{3,\mathrm{sup}}(M) = -5.866 + 11.57M - 6.665M^2 + 1.312M^3 - 0.350\exp[(1-M)/0.012] \quad (10)$$

and

$$C_{1,\mathrm{sup}} = 6.48; \qquad C_{2,\mathrm{sup}} = 8.93; \qquad C_{3,\mathrm{sup}} = 12.21 \quad (11)$$

In the intermediate regime, the drag coefficient is expressed as a nonlinear interpolation between the drag coefficients at $M = M_{\mathrm{cr}}$ and $M = 1$:

$$C_{D,\mathrm{sub}}(Re, M) = C_{D,M_{\mathrm{cr}}}(Re) + [C_{D,M=1}(Re) - C_{D,M_{\mathrm{cr}}}(Re)]\xi_{\mathrm{sub}}(M, Re) \quad (12)$$

where

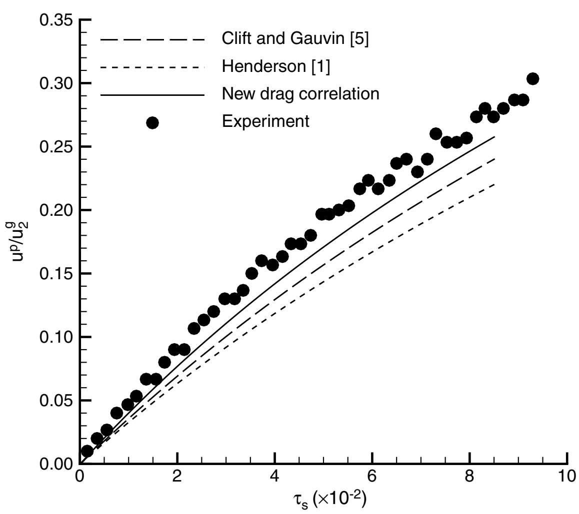
$$\xi_{\mathrm{sub}}(M, Re) = \sum_{i=1}^{3} f_{i,\mathrm{sub}}(M) \prod_{\substack{j\neq i\\ j=1}}^{3} \frac{\log Re - C_{j,\mathrm{sub}}}{C_{i,\mathrm{sub}} - C_{j,\mathrm{sub}}} \quad (13)$$

with

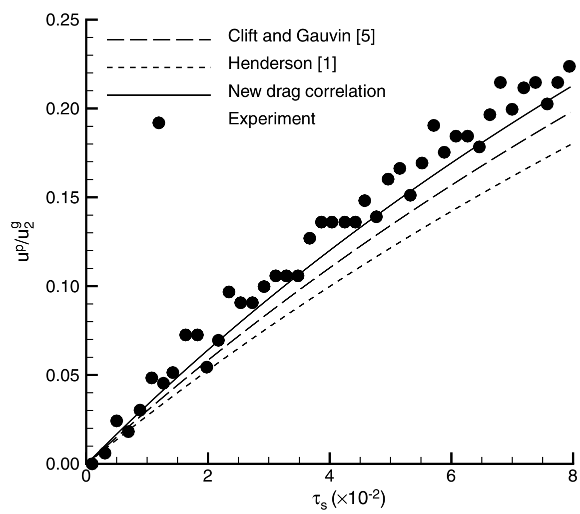
$$f_{1,\mathrm{sub}}(M) = -1.884 + 8.422M - 13.70M^2 + 8.162M^3 \quad (14)$$

$$f_{2,\mathrm{sub}}(M) = -2.228 + 10.35M - 16.96M^2 + 9.840M^3 \quad (15)$$

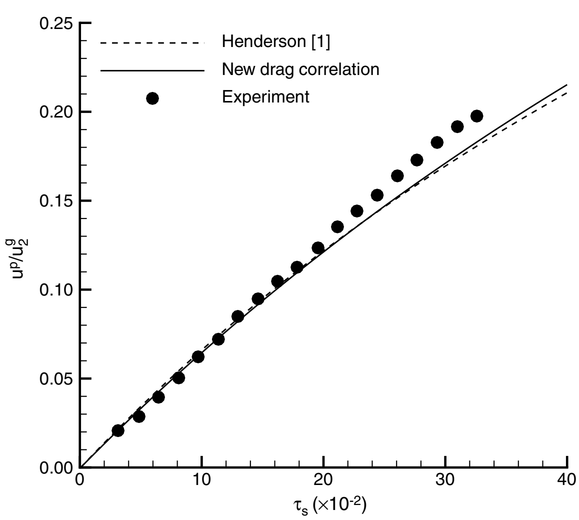
$$f_{3,\mathrm{sub}}(M) = 4.362 - 16.91M + 19.84M^2 - 6.296M^3 \quad (16)$$



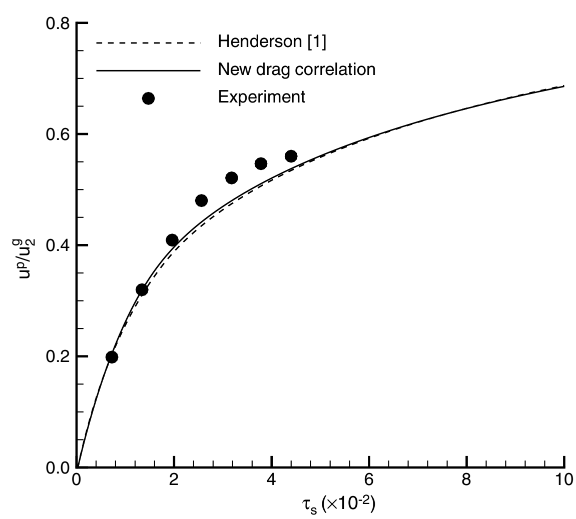
a) Case 148



b) Case 166a



c) Case 181a



d) Case 184a

Fig. 4 Comparison of model with experimental data of Jourdan et al. [4] and Jourdan and Houas [8]. Note the abscissa scaling.

and

$$C_{1,\mathrm{sub}} = 6.48; \qquad C_{2,\mathrm{sub}} = 9.28; \qquad C_{3,\mathrm{sub}} = 12.21 \tag{17}$$

The range of applicability of the improved correlation is $Re \leqslant 2 \cdot 10^5$, $M \leqslant 1.75$, and $Kn < 0.01$. The improved correlation is compared with the data of Bailey and Starr [3] in Fig. 2. Within the range of applicability, the largest deviation between the improved correlation and the data of Bailey and Starr is only 2.5% for $M > 0.6$. This deviation is substantially smaller than those for the correlations of Henderson [1] and Loth [2]. In Fig. 3, a comparison with the earlier data of Goin and Lawrence [6] and May and Witt [7] is shown. It can be seen that, even at lower Reynolds numbers, the new drag-coefficient correlation agrees quite well with experimental data.

III. Validation

We validate the new drag-coefficient correlation using the shock-particle interaction experiments of Jourdan et al. [4]. The measurements of Jourdan et al. are more suitable than others because of the following.

1) Boundary-layer effects are eliminated by hanging the spherical particles from a spider web thread.

2) A large part of the particle trajectory is recorded using multiple shadowgraphs in a single run.

3) Interference between particles is reduced by testing no more than three particles simultaneously.

Table 1 lists the cases that we use to validate our model. The cases include conditions leading to subsonic and supersonic particle Mach numbers behind the shock wave. (See Table 2 in [4] for a complete list of cases and the report by Jourdan and Houas [8] for more details.)

The time history of the particle velocity measured by Jourdan et al. [4] is compared against the velocity computed from

$$m^p \frac{\mathrm{d}u^p}{\mathrm{d}t} = F(t) \tag{18}$$

where m^p is the mass of the particle, u^p is the particle velocity, and $F(t)$ is the drag force on the particle. We assume that the drag force on the particle is given by the quasi-steady drag,

$$F(t) = \tfrac{\pi}{8}\rho_2[u_2^g - u^p(t)]|u_2^g - u^p(t)|(d^p)^2 C_D[Re(t), M(t)] \tag{19}$$

where ρ_2 and u_2^g are the density and velocity of the gas behind the shock (assumed to be constants), d^p is the diameter of the particle, $C_D[Re(t), M(t)]$ is given by Eq. (2), and the relative Reynolds and Mach numbers are

$$Re(t) = \frac{\rho_2|u_2^g - u^p(t)|d^p}{\mu_2} \quad \text{and} \quad M(t) = \frac{|u_2^g - u^p(t)|}{a_2} \tag{20}$$

where μ_2 and a_2 are the dynamic viscosity and speed of sound of the gas behind the shock. Other forces, such as those arising from unsteady effects and gravity, are negligible for the time scales of interest in this study. See Parmar et al. [9,10] for more information on why the inviscid unsteady force can be neglected.

The results are presented in Fig. 4, where the particle velocity u^p normalized by the postshock gas velocity u_2^g is plotted against the nondimensional time $\tau_s = u_s t/d^p$. (Note that $\tau_s = 1$ corresponds to the time required for the shock wave to propagate over the particle.) As can be seen from Figs. 4a and 4b, the particle velocity is predicted quite accurately with the new drag-coefficient correlation for cases 148 and 166a. Because the relative Mach numbers are subsonic and mostly subcritical, the difference between the results obtained with the standard and the new drag-coefficient correlations are relatively small. The standard-drag-coefficient correlation leads to better results than Henderson's [1] correlation. The explanation for this result is that Henderson's correlation underpredicts the standard-drag-coefficient curve for the range of Mach and Reynolds numbers encountered in cases 148 and 166a (see Fig. 1a). Figures 4c and 4d show results for cases 181a and 184a, where the relative Mach number after the passage of the shock wave over the particle is supercritical. The results obtained with both the new correlation and Henderson's correlation agree well with the experimental data.

Acknowledgments

The authors gratefully acknowledge support by the National Science Foundation under grant number EAR0609712. The authors thank Georges Jourdan and Eric Loth for helpful discussions.

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Associate Editor